DL 2 – Heat Capacity and Heat of Vaporization

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In order to properly cover the material from Activity 1.2.1 from DL2, I have made this document to cover the quantitative relationships governing different forms of energy change.

The idea is this: given a certain substance, how much does the temperature/mass change when we put in a specific amount of energy? We can answer this in the context of our 3-Phase Model by looking more closely at our phase diagrams.



Figure 1: The process shown is a pure gas rising in temperature as heat is transferred to the generic substance. T_{VP} and T_{MP} are the *vaporization* and *melting* temperatures respectively.

In the above graph, we have some arbitrary gas in pure phase that is increasing in temperature as we add energy. Notice that the substance follows a linear path - if you remember your slope equation for a line, **rise/run**, we can write

$$\frac{\Delta T}{\Delta E} = \text{slope constant} \equiv \frac{1}{C}$$

where I define 1/C to be a constant, the slope. Furthermore, rewriting this gives

$$\Delta E = C\Delta T$$

In addition, if the only energy being transferred to our system is through heat, we have that

$$\Delta E_{th} = Q = C\Delta T$$

We now have a relationship between the *change in energy* and the *change in temperature*. They are linearly related by the constant C, which we can call the **heat capacity**.

How do we interpret this constant? We see from our equation that the units of C have to be Joules/Kelvin in order to give energy. So, we can think of heat capacity as *the energy* needed to raise our **specific amount** of substance by 1° K. Heat capacity depends on

- 1. What kind of substance we are looking at.
- 2. How much of that substance there is.

Now, one might say: I don't like working with a constant that depends on how much of this substance we have! Well, let's create a constant called *specific heat*, given by "little c". Specific heat is how much energy needed to heat a unit mass of a particular substance. The relationship between C and c is given by

$$c = \frac{C}{m} \implies C = mc$$

Note that if we wanted to use *moles* instead of *mass* to measure our amount, we can just switch the m with n (the number of moles).

Let's look at a different process in the phase diagram now – say one of the phase transition regions. In the diagram below, let's say our substance is changing from a liquid to a gas.



Figure 2: Here, a substance is changing phase; it is in *mixed phase*, going from a liquid to a gas as heat is transferred to the system.

If we are in mixed phase, what is true? We know the following:

- The temperature is not changing, and the system is in thermal equilibrium.
- The change in energy is not in E_{th} , but rather E_{bond} .
- As bonds break, we know that either a *phase transition* or *chemical reaction* is occurring, so the amount of a particular phase or substance must be going down while another goes up (that is, m_l decreases while m_q increases).

So, given this information, we can make a formula for E_{bond} like we did for E_{th} . But look back at our new phase diagram – there is no temperature change! So, we cannot just use the graph because ΔE_{bond} does not depend on ΔT .

What was our indicator for E_{th} change? ΔT . We observed that ΔE is related to ΔT by a constant. What is our indicator for E_{bond} change? Δm ! So it might not be crazy to think that there is also a linear relationship between ΔE_{bond} and Δm , as in

$$\Delta E_{bond} = h \,\Delta m$$

where h is just some constant.

Now, we know that E_{bond} is the energy associated with breaking bonds, however there is also an energy associated with *breaking bonds for a particular configuration of molecules*, kind of like how we have a unique heat capacity for a given substance! Let's call this quantity ΔH , the **heat of vaporization/melting**, or in chemical reactions **heat of formation**.

If we assume that ΔH is analogous to heat capacity, then our constant $h = \Delta H$ in our above equation, so

$$\Delta E_{bond} = \Delta H \,\Delta m$$

Note: ΔH gives the heat of vaporization/melting for an *entire phase change*, so is the amount of energy needed to *completely vaporize/melt a unit of the substance* (units of ΔH are Joules/kilograms).

In conclusion, the following equations are true for cases within our 3-Phase Model of Pure Substances, and now include signs to maintain generality:

$$\Delta E_{th} = \pm |C\Delta T|, \quad \Delta E_{bond} = \pm |\Delta H \,\Delta m|$$

Remember: either of these energy changes are *positive* if heat or work is going *into the* system. Otherwise, the signs are negative, as energy is *leaving* the system.